Period 3, Sept 23, 2024

23. $y = \dfrac{4x^{3/2}}{x}$

$y = 4x^{\frac{3}{2}-1} = 4x^{\frac{1}{2}}$

$\dfrac{dy}{dx} = 4 \cdot \frac{1}{2} \cdot x^{\frac{1}{2}-1} = 2 \cdot x^{-\frac{1}{2}} = \dfrac{2}{\sqrt{x}} \cdot \dfrac{\sqrt{x}}{\sqrt{x}} = \dfrac{2\sqrt{x}}{x}$

29. $f(x) = \dfrac{3x-1}{\sqrt{x}} = \dfrac{3x}{\sqrt{x}} - \dfrac{1}{\sqrt{x}} = 3x^{1-\frac{1}{2}} - 1x^{-\frac{1}{2}} = 3x^{\frac{1}{2}} - 1x^{-\frac{1}{2}}$

$F'(x) = 3 \cdot \frac{1}{2} x^{\frac{1}{2}-1} - 1 \cdot -\frac{1}{2} \cdot x^{-\frac{1}{2}-1}$

$F'(x) = \dfrac{3}{2} x^{-\frac{1}{2}} + \dfrac{1}{2} \cdot x^{-3/2} = \dfrac{3 \cdot x}{2\sqrt{x} \cdot x} + \dfrac{1}{2x\sqrt{x}} = \dfrac{3x+1}{2x\sqrt{x}}$

$F(x) = \dfrac{3x-1}{\sqrt{x}}$

$\dfrac{x^5 - x^3}{x^2} = \dfrac{x^2(x^3 - x)}{x^2} = x^3 - x$

$F'(x) = \dfrac{3 \cdot \sqrt{x} - (3x-1) \cdot \frac{1}{2\sqrt{x}}}{(\sqrt{x})^2} = \dfrac{\frac{3\sqrt{x} \cdot 2\sqrt{x}}{1 \cdot 2\sqrt{x}} - \frac{3x-1}{2\sqrt{x}}}{x} = \dfrac{\frac{6x}{2\sqrt{x}} - \frac{3x-1}{2\sqrt{x}}}{x}$

$\dfrac{\frac{6x-3x+1}{2\sqrt{x}}}{\frac{x}{1}} = \dfrac{3x+1}{2\sqrt{x}} \cdot \dfrac{1}{x} = \dfrac{3x+1}{2x\sqrt{x}}$

54. $h(\theta) = 5\theta \sec\theta + \theta \tan\theta$

$h'(\theta) = 5 \cdot \sec\theta + 5\theta \cdot \sec\theta\tan\theta + 1 \cdot \tan\theta + \theta \cdot \sec^2\theta$

$5\sec\theta + \theta\sec^2\theta + 5\theta\sec\theta\tan\theta + \tan\theta$

~~$\sec\theta(5 + \theta\sec\theta) + \tan\theta(5\theta\sec\theta + 1)$~~

53. $y = 2x \sin x + x^2 \cos x$

$\dfrac{dy}{dx} = 2(\sin x) + 2x(\cos x) + 2x(\cos x + x^2(-\sin x)$

$\dfrac{dy}{dx} = 2\sin x + 2x\cos x + 2x\cos x - x^2\sin x$

$= 2\sin x + 4x\cos x - x^2\sin x$

52. $f(x) = \sin x \cos x$

$F'(x) = \cos x \cdot \cos x + \sin x(-\sin x)$

$\cos^2 x - \sin^2 x$

$\cos 2x$

$1 - 2\sin^2 x$

$2\cos^2 x - 1$

$\frac{2\cdot 1}{2\cdot 3} + \frac{1\cdot 3}{2\cdot 3} = \frac{5}{6}$

30. $f(x) = \sqrt[3]{x}(\sqrt{x} + 3)$

$F(x) = x^{\frac{1}{3}}(x^{\frac{1}{2}} + 3)$

$F(x) = x^{\frac{1}{3}+\frac{1}{2}} + 3x^{\frac{1}{3}}$

$F(x) = x^{5/6} + 3x^{\frac{1}{3}}$

$F'(x) = \frac{5}{6}x^{5/6-1} + 3\cdot\frac{1}{3}x^{\frac{1}{3}-1}$

$\frac{5}{6}x^{-\frac{1}{6}} + 1x^{-\frac{2}{3}}$

$\frac{5\sqrt[6]{x^5}}{6\sqrt[6]{x}\cdot\sqrt[6]{x^5}} + \frac{6\cdot 1\sqrt[3]{x}}{\sqrt[3]{x^2}\cdot\sqrt[3]{x}\cdot 6}$

$\frac{5}{6\sqrt[6]{x}\cdot\sqrt[6]{x^5}} = \frac{5\sqrt[6]{x^5}}{6\sqrt[6]{x^1\cdot x^5}} = \frac{5\sqrt[6]{x^5}}{6\sqrt[6]{x^6}}$

$\frac{5\sqrt[6]{x^5}}{6x}$

30. $f(x) = \sqrt[3]{x}(\sqrt{x} + 3) = x^{\frac{1}{3}}(x^{\frac{1}{2}} + 3)$

$F'(x) = \frac{1}{3}\cdot x^{-\frac{2}{3}}(x^{\frac{1}{2}} + 3) + x^{\frac{1}{3}}\left(\frac{1}{2}x^{-\frac{1}{2}}\right)$

$\frac{1}{3}x^{-\frac{2}{3}\cdot\frac{2}{2}\cdot\frac{3}{3}} + x^{-\frac{2}{3}} + \frac{1}{2}\cdot x^{\frac{2\cdot 1}{2\cdot 3} - \frac{1\cdot 3}{2\cdot 3}}$

$\frac{1}{3}x^{-\frac{1}{6}} + x^{-\frac{2}{3}} + \frac{1}{2}x^{-\frac{1}{6}} = \frac{1\cdot 2\sqrt[6]{x^5}}{2\cdot 3\sqrt[6]{x}\cdot\sqrt[6]{x^5}} + \frac{\sqrt[3]{x}}{\sqrt[3]{x^2}\cdot\sqrt[3]{x}} + \frac{1\cdot 3\sqrt[6]{x^5}}{3\cdot 2\sqrt[6]{x}\cdot\sqrt[6]{x^5}}$

$\frac{2\sqrt[6]{x^5}}{6x} + \frac{6\cdot\sqrt[3]{x}}{6\cdot x} + \frac{3\sqrt[6]{x^5}}{6x}$

$\frac{2\sqrt[6]{x^5} + 6\sqrt[3]{x} + 3\sqrt[6]{x^5}}{6x} = \frac{5\sqrt[6]{x^5} + 6\sqrt[3]{x}}{6x}$

17. $f(x) = x \cos x$ $\quad c = \frac{\pi}{4}$

$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$F'(x) = 1 \cdot \cos x + x \cdot (-\sin x)$

$F'(x) = \cos x - x \sin x$

$F'\left(\frac{\pi}{4}\right) = \cos\frac{\pi}{4} - \frac{\pi}{4} \cdot \sin\frac{\pi}{4}$

$\frac{\sqrt{2}}{2} - \frac{\pi}{4} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2} \cdot 4}{2 \cdot 4} - \frac{\pi\sqrt{2}}{8} = \frac{4\sqrt{2} - \pi\sqrt{2}}{8}$

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49. $y = -\csc x - \sin x$ $= -\csc x - \sin x$

$\frac{dy}{dx} = -\left(-\csc x \cot x\right) - (\cos x)$

$\csc x \cot x - \cos x$

$\frac{1}{\sin x} \cdot \frac{\cos x}{\sin x} - \frac{\cos x \sin^2 x}{1 \cdot \sin^2 x} = \frac{\cos x - \cos x \sin^2 x}{\sin^2 x} = \frac{\cos x (1 - \sin^2 x)}{\sin^2 x}$

$\frac{\cos^3 x}{\sin^2 x} = \cos x \cot^2 x = \frac{\cos x \cdot \cos^2 x}{\sin^2 x}$

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40. $f(\theta) = (\theta + 1)\cos\theta$

$F'(x) = 1 \cdot \cos\theta + (\theta + 1) \cdot (-\sin\theta)$

$F'(x) = \cos\theta - \theta\sin\theta - \sin\theta$

51. $f(x) = x^2 \tan x$

$F'(x) = 2x \cdot \tan x + x^2 \cdot \sec^2 x$

$F'(x) = \frac{2x \sin x \cos x}{\cos x \cdot \cos x} + \frac{x^2 \cdot 1}{\cos^2 x}$

$= \frac{2 \cdot x \cdot \sin x \cos x + x^2}{\cos^2 x}$

$\frac{x(\sin 2x + x)}{\cos^2 x}$ $= \frac{x(2\sin x \cos x + x)}{\cos^2 x}$

$$h(x) = \frac{F(x)}{g(x)}$$

$$h'(x) = \frac{F'(x)\cdot g(x) - F(x)\cdot g'(x)}{(g(x))^2}$$

$$F(x) = \tan x = \frac{\sin x}{\cos x}$$

$$F'(x) = \frac{\cos x \cdot \cos x - \sin x(-\sin x)}{(\cos x)^2}$$

$$F'(x) = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$$

$$h(x) = \frac{(3x^7-5)(7x^2+1)}{x^9+6x^2}$$

$$h'(x) = \frac{\left[21x^6(7x^2+1) + (3x^7-5)(14x)\right](x^9+6x^2) - (3x^7-5)(7x^2+1)(9x^8+12x)}{(x^9+6x^2)^2}$$